December 2011

MADALGO seminars by Kasper Green Larsen, Aarhus University

On Range Searching in the Group Model and Combinatorial Discrepancy

Abstract:

In this talk we establish an intimate connection between dynamic range searching in the group model and combinatorial discrepancy. Our result states that, for a broad class of range searching data structures (including all known upper bounds), it must hold that $t_u t_q = \Omega(\mathrm{disc}^2/\lg n)$where t_u is the worst case update time, t_q the worst case query time and disc is the combinatorial discrepancy of the range searching problem in question. This relation immediately implies a whole range of exceptionally high and near-tight lower bounds for all of the basic range searching problems. We list a few of them in the following:

- For halfspace range searching in d-dimensional space, we get a lower bound of $t_u t_q = \Omega(n^{1-1/d}/\lg n)$. This comes within a $\lg n \lg\lg n$ factor of the best known upper bound.
- For orthogonal range searching in d-dimensional space, we get a lower bound of $t_u t_q = \Omega(\lg^{d-2+\mu(d)} n)$, where $\mu(d) > 0$ is some small but strictly positive function of d.
- For ball range searching in d-dimensional space, we get a lower bound of $t_u t_q = \Omega(n^{1-1/d}/\lg n)$.

We note that the previous highest lower bound for any explicit problem, due to Pătraşcu [STOC'07], states that $t_q = \Omega((\lg n/\lg(\lg n + t_u))^2)$, which does however hold for a less restrictive class of data structures.

Our result also has implications for the field of combinatorial discrepancy. Using textbook range searching solutions, we improve on the best known discrepancy upper bound for axis-aligned rectangles in dimensions $d \geq 3$.

This work was presented at FOCS'11, where it received the Best Student Paper Award.